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ABSTRACT

In the present work the concentration of chromite fines by a multigravity separator (MGS) has been studied. The main goal of the experimental work has been the evaluation of the influence of the main MGS process parameters in the enrichment of an Albanian ore. The experimental tests have involved the design of techniques using factor analysis as a support for full or fractional factorial experiments. Therefore, factorial experiments have been performed in order to find the main and interaction effects of the investigated factors on MGS performance: shake frequency, shake amplitude, tilt angle, washwater flow rate, and size fraction. The factor analysis supported factorial experiments in order to reduce the responses considered in the analysis of the variance (ANOVA). This statistical method has been used to identify the truely independent variables of the process. A very strong correlation has been found among Cr_2O_3 grade and the other elements. For this reason the only responses of the process were the recovery of Cr_2O_3 in the concentrate and its grade. The results obtained revealed the technical feasibility of the separation of chromite fines by MGS and the advantages that can be obtained using statistical methods in the design of experiments for the study of multifactor processes.

Key Words. Factorial experiments; Factor analysis; Separation; Chromite fines; Wet operations; Multigravity separator

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INTRODUCTION

The separation of fine and ultra fine ores gives rise to many problems. As a result, conventional gravity mineral processing equipment is usually inefficient and therefore not cost-effective. Inevitably there is a substantial loss of valuable minerals in the slime streams from any mineral processing plant. In fact, mining and milling operations in general involve a large tonnage of lower quality ores which are usually characterized by their very small liberation size. The need for fine grinding in conjunction with improved beneficiation technology is becoming increasingly important. Samasundaran (1) reported that more than 25% of the mineral value is lost as slimes during the processing of many ores.

Improvements in gravity separation techniques for fine mineral processing in recent years have been directed toward improving production costs through performance optimization by a better understanding of the behavior of mineral particles in the stream or scale up techniques.

Wet gravity separation methods have been developed in recent years for the processing and separation of fine particles. The recovery of high-value minerals in fine-particle form is a very difficult problem, especially when the separation has to be achieved by means of the wet gravity approach. Indeed, with conventional methods the bulk of the values that occur in fines are lost.

Various treatment methods and machines have been developed in an attempt to treat fine ores by the wet gravity approach. From the different technologies developed for the recovery of valuable metals, a very important role can be played by the introduction of the Mozley Multi-Gravity Separator (MGS). The principle of the MGS may be visualized as a rolling of the horizontal surface of a conventional shaking table into a drum, then rotating it so that a gravitational pull many times the normal can be exerted on the mineral particles as they flow in the water layer across the surface (see Fig. 1). This machine is able to recover particles down to 1 μm in diameter with at least a 1.0 specific gravity unit of difference between the two main mineralogical forms. The limit of conventional gravity concentrators is about 5 μm and higher (2). However, this limit is more theoretical that practical from a metallurgical point of view.

A problem of particular interest is linked to the recovery of chromite fines by the gravity approach. In these cases some interesting results in treatment have been obtained by the MGS (2–7).

The concentration of chromite from fine ores is related to the process parameters that can be set in the MGS. For this reason it is essential to study the influence of these factors on the recovery and, in particular, on the concentration of chromite in the treatment process of fine ores.

An initial preliminary screening study, reported elsewhere (7), was carried out by 2^n full-factorial experiments (8) in order to identify the main operating conditions in the separation of chromite fines. In that paper the study was car-

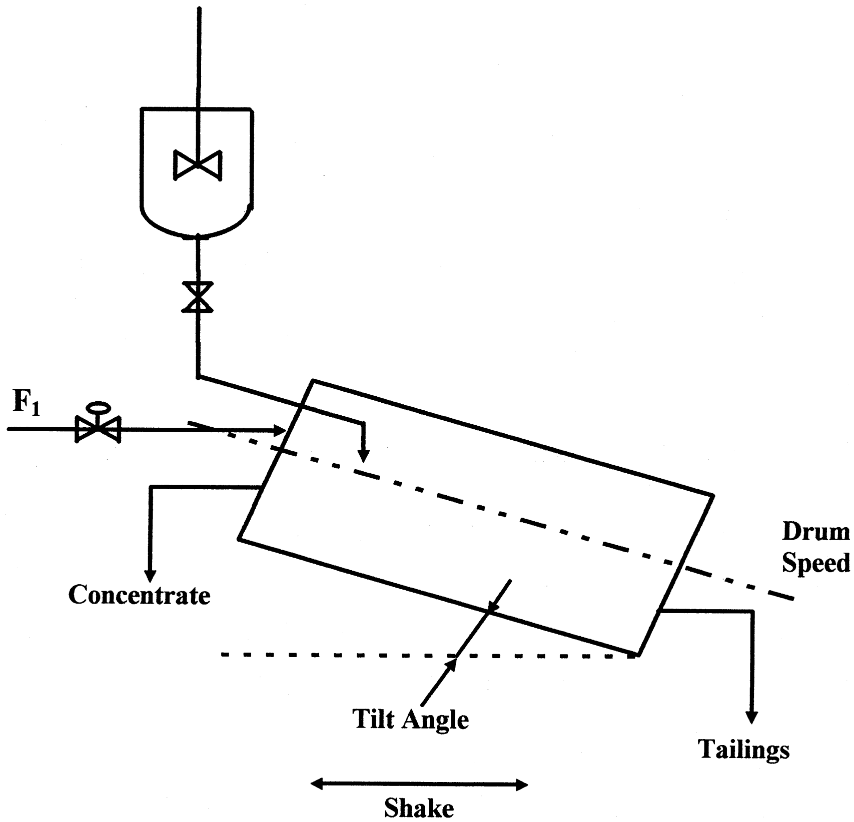

FIG. 1 Schematic representation of the experimental apparatus (MGS) used in the experimental tests.

ried out with the same mineral but without control of its size fraction. The liberation degree characteristic and the textural analysis of the mineral are reported there (7): the liberation degree of <100 μm chromite was 100% (d_{80} of 100 μm). A study using the surface response methodology was also carried out (9).

This work was performed by considering the same MGS process parameters yet with a different range of factor levels and considering two different size mineral fraction in the experimentation in order to evaluate the influence of the investigated factors on the Cr_2O_3 grade and the recovery of the concentrate and of the tailings. As reported elsewhere (7), the best operating conditions are those capable of producing elevated Cr_2O_3 contents with high recovery and a low SiO_2 content in the concentrate [therefore with elevated values of the enrichment composition ratio, defined as $R = (Cr_2O_3 \text{ grade})/(SiO_2 \text{ grade})$ in the concentrate] (7).

In general, the factors that increase the recovery usually tend to depress the selectivity of the separation process and therefore R.

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The aim of this work has been to determine the effects (by fractional factorial experiments) of the MGS operative factors on the Cr_2O_3 grade and recovery from an Albanian ore by considering different size fractions. The factors considered in the study were: shake frequency and amplitude, tilt angle, washwater flow rate, drum speed, and size fraction. Factor analysis (FA) of the responses considered in the factorial experiment has been used in order to evaluate the intrinsic dependence among the factors. The aim of the optimization strategy used in this work was:

1. To design the experimental tests (using fractional factorial design)
2. To evaluate by FA the real independent responses evaluated for each experimental test
3. To perform an analysis of the experimental results by ANOVA to determine the significant factors influencing the separation process

A simulation procedure has been proposed to evaluate the performance of MGS under study in the separation of fines.

MATERIALS AND METHODS

Characteristic of the Ore

The ore was ground for 35 minutes in a steel laboratory rod mill to obtain a d_{80} of 100 μm (80% of the mineral is smaller than 100 μm). A complete mineralogical analysis of the chromite sample is reported elsewhere (7). In particular, chromium-iron-magnesium spinel $(Mg,Fe)(Cr,Al,Fe)_2O_4$, serpentine $Mg_e(Si_4O_{10})(OH_8)$, and magnesic olivine $(Mg,Fe)_2SiO_4$ are present in the ore. Pilot tests have been carried out with this mineral fractionated into fractions of two different sizes; the main chemical compositions of these fractions is are reported in Table 1.

Experimental Procedures

The experimental tests were carried out in a pilot-scale MGS (at the CNR-Mineral Processing Institute, Rome, Italy). A complete description of the operating principle of the machine has been reported elsewhere (7, 9).

TABLE 1
Main Characteristic of Two Fractions of the Chromite Ore

Size fraction (μm)	Cr_2O_3 grade (%)	R_0
20–40	10.4 ± 0.9	0.39 ± 0.2
150–210	25.0 ± 1.5	1.49 ± 0.1

The pilot-scale MGS used in this work consists of a 1.65 m long, 0.5 m diameter drum which rotates clockwise at variable speeds between 130 and 250 rpm. It is capable of generating a centrifugal force equivalent to a gravitational pull between 6 and 24*g* at the drum surface. A sinusoidal shake with a variable amplitude between 12 and 25 mm in the axial direction and a frequency (or speed) variable between 4 and 6 cycles per second (cps) is superimposed on the axial motion of the drum.

Batch tests were carried out using the desired settings of the operative parameters according to a factorial design. These tests were started with the following settings: the shake amplitude was set between 10 and 20 mm; the shake frequency was adjusted between 4.0 and 5.7 cps; the tilt angle (meaning the angle between the drum axis and horizontal) was preset between 2 and 4°.

In general, 1000 g of dry sample was mixed with water to obtain a suspension of 33% solids w/w. The solid was kept in suspension during the experimental test by a mechanical stirrer. The suspension was fed to the MGS at a constant rate and the feeding time was always around 2 minutes. The feeding point of the water was close to the concentrate discharge. Figure 1 is a schematic representation of the experimental apparatus.

Following the separation process, two fractions of the initial mineral were obtained: a concentrate and a tail fraction. These two fractions were analysed by x-ray diffraction (XRD), x-ray fluorescence (XRF), scanning electron microscopy (SEM) with energy, a dispersive analyzer (EDS), and chemical analysis by ICP to determine the composition of the concentrate and of the sterile fractions (7). A material balance was carried out in order to check the experimental tests.

Statistical Methods

Planning of the experimental runs was carried out using a full or fractional factorial design. This methodology is very helpful in both the experimental planning and the statistical interpretation of the experimental results (8, 10) (by ANOVA analysis). In this manner it is possible to arrange an orthogonal experimental plane in which it is possible to evaluate independently both the main effect and the interaction among the factors investigated for a given response. The Yates' notation was used to name each experimental condition (treatment) (8). In general, Cr_2O_3, SiO_2, Fe_2O_3, MgO, and Al_2O_3 grades and recoveries have been experimentally evaluated for each treatment as responses to the separation process.

Factor analysis (noted as FA) (11) permitted to establishment of which of these responses were the only real independent variables.

The experimental results were elaborated by using ANOVA in which it is possible to evaluate if the effect and the interaction among the investigated factors are significant with respect to experimental error. In general, an effect is statistically significant if its significant level is larger than 95%. The effect of a factor is the change in response produced by a change in the level of the factor. When the effect of a factor depends on the level of another factor, the two factors are said to interact.

According to the principles of factorial design, all the experimental tests were randomized. Bartlett's test was performed in all factorial experiments to check the variance equality (10).

RESULTS

Factorial Experiment

The optimization study was carried out by means of fractional and full factorial experiments used with a typical sequential approach (12). The full and fractional factorial experiments are methods of the design of experiments (DOE) in which a statistical analysis (carried out by ANOVA) is performed to evaluate the significance of the main and interaction effects as evaluated from the experimental results. In particular, they are used when several factors have to be studied in order to determine their main effects and interactions. It is possible to show any advantage obtained when a factorial design is used over in experimental work with respect to the method of "changing a factor at a time" (10).

In the present work, six factors were taken in consideration to evaluate their main and interaction effects on the responses of the process under study (component grades and their recoveries in the concentrate fraction) in order to study the separation process of chromite fines of industrial interest. In other words, the main goal has been to establish the best set of process parameters that could be set in the MGS to obtain the best enrichment in Cr_2O_3 in the concentrate fraction with acceptable recoveries. Shake frequency (A), shake amplitude (B), tilt angle (C), washwater flow rate (D), drum speed (E), and size fraction (F) were the factors under study.

A complete factorial experiment, in which all the possible combinations of all the levels of different factors are investigated, will involve a large number of trials if we consider a number of factors larger than 5, as in the case of this study. In these cases it is possible to show that a minor number of experiments can be planned by taking advantage of fractional factorial design (10).

The philosophy of this kind of experimental design is linked to the lack of interest that scientists have for high-order interactions. The fractional design can be performed by confounding the contrast obtained to evaluate a high-order interaction with the effect of a given factor (8). With these considerations it is possible to obtain the experimental planning shown in Table 5, where the effect of factor F is confused with the interaction ABCDE (F = ABCDE). In this manner, 32 treatments can be carried out to study 6 factors at two levels

TABLE 2
Factors and Levels Investigated in the a Half 2_V^{6-1} Fractional Factorial Design

Factors	Levels	
	−	+
A Shake frequency (cps)	4.0	5.7
B Shake amplitude (mm)	10.0	20.0
C Tilt angle (°)	2.0	4.0
D Washwater flow rate (L/min)	2.0	6.0
E Drum speed (rpm)	150.0	220.0
F Size distribution (μm)	0–20	150–210

instead of 64, performing an experimental design of resolution V: more detailed information about the experimental design technique can be found in specialist literature (8, 10).

To establish the influence of these process parameters on the performance of MGS, a half 2_V^{6-1} fractional factorial experiment was performed (10). The performance of MGS has been established in the separation of chromite fine under study. In particular the main goal of this study was to determine the influence of shake frequency, shake amplitude, tilt angle, washwater flow rate, drum speed, and size fraction of the mineral on the separation of this mineral.

In Table 2 the factors and the levels investigated in the fractional factorial experiments are shown: the 32 treatments of this fractional factorial plane are shown in Table 3. Yate's notation was used to name each treatment (8).

For example, treatment *ab* is the experimental run in which the factor A and B are set at their highest levels whereas factors C, D, E, and F are at their lowest levels (see Table 2). In each treatment the responses of the process, Cr_2O_3, Fe_2O_3, SiO_2, Al_2O_3, and MgO grades and recoveries were determined for the concentrate and the sterile fraction.

Data Analysis

Table 4 and 5 show the experimental results. The component concentrations in the concentrate fraction are reported in Table 4; Table 5 reports the relative component recoveries. Similar results were obtained for the sterile fraction (data not shown in tables but reported in Fig. 3).

All the responses (grades into the concentrate fraction) were analyzed by factor analysis (11) to determine the possible correlation among the responses of the process considered in the factorial experiment (i.e., Cr_2O_3, SIO_2 grade, etc.). In fact, in order to optimize the recovery of Cr_2O_3 in the concentrate and

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TABLE 3
Treatments of the Fractional Factorial Design: Defining Contrast I, ABCDE

		Factors					
No.	Treatment	A	B	C	D	E	F
1	(1)	4.0	10.0	2.0	2.0	150	0–20
2	af	5.7	10.0	2.0	2.0	150	150–210
3	bf	4.0	20.0	2.0	2.0	150	150–210
4	ab	5.7	20.0	2.0	2.0	150	0–20
5	cf	4.0	10.0	4.0	2.0	150	150–210
6	ac	5.7	10.0	4.0	2.0	150	0–20
7	bc	4.0	20.0	4.0	2.0	150	0–20
8	abcf	5.7	20.0	4.0	2.0	150	150–210
9	df	4.0	10.0	2.0	6.0	150	150–210
10	ad	5.7	10.0	2.0	6.0	150	0–20
11	bd	4.0	20.0	2.0	6.0	150	0–20
12	abdf	5.7	20.0	2.0	6.0	150	150–210
13	cd	4.0	10.0	4.0	6.0	150	0–20
14	acdf	5.7	10.0	4.0	6.0	150	150–210
15	bcdf	4.0	20.0	4.0	6.0	150	150–210
16	abcd	5.7	20.0	4.0	6.0	150	0–20
17	ef	4.0	10.0	2.0	2.0	220	150–210
18	ae	5.7	10.0	2.0	2.0	220	0–20
19	be	4.0	20.0	2.0	2.0	220	0–20
20	abef	5.7	20.0	2.0	2.0	220	150–210
21	ce	4.0	10.0	4.0	2.0	220	0–20
22	acef	5.7	10.0	4.0	2.0	220	150–210
23	bcef	4.0	20.0	4.0	2.0	220	150–210
24	abce	5.7	20.0	4.0	2.0	220	0–20
25	de	4.0	10.0	2.0	6.0	220	0–20
26	adef	5.7	10.0	2.0	6.0	220	150–210
27	bdef	4.0	20.0	2.0	6.0	220	150–210
28	abde	5.7	20.0	2.0	6.0	220	0–20
29	cdef	4.0	10.0	4.0	6.0	220	150–210
30	acde	5.7	10.0	4.0	6.0	220	0–20
31	bcde	4.0	20.0	4.0	6.0	220	0–20
32	abcdef	5.7	20.0	4.0	6.0	220	150–210

to minimize the other components present in the mineral, it would be appropriate to evaluate the influence of the factors tested for the different selected responses (i.e., component grades). This means performing the ANOVA for each response and determining the influence of the factors investigated on the response under examination (i.e., Cr_2O_3 grade). If the responses considered in the factorial experiment are linked together by an internal correlation, it will not possible to optimize the separation of Cr_2O_3 with respect to the other

TABLE 4
Component Grades in the Concentrate

No.	Weight of concentrate (%)	Fe_2O_3 (%)	Cr_2O_3 (%)	Al_2O_3 (%)	SiO_2 (%)	MgO (%)	Cr/Si ratio
1	3.90	18.39	47.19	5.35	4.82	24.91	9.79
2	66.74	15.10	34.90	4.40	12.40	30.30	2.81
3	57.82	15.29	35.79	3.99	11.80	31.89	3.03
4	0.32	13.13	22.76	3.22	18.80	33.90	1.21
5	70.47	14.55	33.20	3.59	13.07	35.96	2.54
6	0.69	20.10	51.96	6.38	2.15	18.61	24.17
7	0.57	19.26	49.47	6.07	3.44	21.40	14.38
8	0.00	15.13	42.00	4.26	10.90	28.99	3.85
9	57.56	15.26	36.17	4.06	10.81	29.84	3.35
10	1.16	19.24	49.81	6.03	3.46	21.66	14.40
11	0.01	20.00	53.00	5.00	3.40	15.00	37.86
12	2.70	15.75	40.90	4.41	10.04	29.46	4.07
13	0.33	19.68	52.40	6.74	1.53	16.47	34.25
14	23.83	17.17	45.48	5.24	6.18	25.73	7.36
15	22.86	16.70	43.48	4.92	6.90	25.44	6.30
16	0.02	18.00	53.00	5.00	3.00	14.70	37.86
17	94.84	13.35	26.69	3.14	17.54	36.20	1.52
18	31.39	11.20	16.30	2.67	22.47	35.28	0.73
19	42.11	10.53	13.51	2.36	24.73	35.66	0.55
20	83.61	13.78	29.09	3.68	14.65	31.53	1.99
21	38.08	11.17	15.83	2.71	23.08	35.82	0.69
22	91.57	13.90	29.16	3.33	15.80	34.96	1.85
23	90.18	13.85	29.50	3.39	15.60	34.65	1.89
24	28.36	13.78	29.09	3.68	14.65	31.53	1.99
25	30.03	11.66	18.20	2.80	21.02	34.51	0.87
26	88.32	16.96	29.80	3.44	15.22	33.88	1.96
27	85.60	13.32	27.66	3.19	15.24	31.65	1.81
28	13.76	14.11	27.48	3.29	16.10	34.92	1.71
29	90.16	13.22	26.89	3.21	15.84	32.39	1.70
30	26.69	11.80	18.83	2.77	21.23	35.80	0.89
31	27.99	12.12	19.24	2.90	21.20	35.86	0.91
32	72.66	14.83	32.99	3.81	13.81	34.20	2.39

components because of these correlation. For this reason a factor analysis was carried out to consider the component grades of the concentrate and sterile fractions before performing the ANOVA for each component present in the mineral.

Factor analysis is widely used to reduce the dimensions of the original variables, especially when they are highly correlated. The objective of factor analysis is to take p variables X_1, X_2, . . . X_p (i.e., Cr_2O_3, Fe_2O_3, SiO_2,

TABLE 5
Component Recoveries in the Concentrate

No.	Fe_2O_3 (%)	Cr_2O_3 (%)	SiO_2 (%)	Al_2O_3 (%)	MgO (%)
1	7.26	17.78	0.75	7.75	3.08
2	76.93	86.40	49.56	81.80	63.08
3	67.74	78.00	40.53	69.49	56.02
4	0.42	0.70	0.24	0.41	0.33
5	80.86	91.28	51.83	84.05	81.24
6	1.41	3.66	0.06	1.96	0.38
7	1.10	2.79	0.07	1.53	0.35
8	0.10	0.14	0.05	0.11	6.6
9	70.72	84.77	35.17	76.12	52.68
10	2.28	5.90	0.16	2.83	0.77
11	0.02	0.10	0.00	0.02	0.00
12	3.00	3.47	1.67	3.39	2.25
13	0.66	1.72	0.02	1.03	0.16
14	31.47	41.64	8.83	33.84	19.13
15	29.28	37.81	9.13	34.93	17.10
16	0.02	0.10	0.00	0.02	0.00
17	97.47	99.38	91.25	97.64	94.83
18	35.76	49.92	27.90	35.36	33.46
19	46.43	60.39	39.28	47.10	44.65
20	89.87	95.64	71.84	91.30	80.75
21	43.14	58.52	33.69	46.91	39.97
22	95.77	98.90	84.71	96.19	91.17
23	94.69	98.24	82.41	95.83	89.43
24	40.71	62.98	16.52	44.79	24.56
25	35.99	53.69	24.39	39.48	31.07
26	95.35	98.91	78.56	94.65	87.28
27	92.70	98.49	74.55	93.84	83.73
28	19.91	37.29	8.40	21.06	14.23
29	95.20	99.12	82.17	95.20	88.61
30	32.10	49.37	21.58	34.71	28.18
31	34.41	52.28	22.90	36.40	29.93
32	82.51	91.53	56.34	86.51	70.40

Al_2O_3, and MgO grades) and find a combination of these to produce indices (factors or latent variables) Z_1, Z_2, linear, Z_p that are orthogonal in the new reference system. The lack of correlation is a useful property because it means that the indices are measuring a different "dimension" in the data. Two terms must be used to describe the results from factor analysis: score and loading. The score is the projection of observation on the corresponding principal component: The loading is the coefficient of each original variables on them (13).

The important mathematical steps for performing factor analysis are as follows. From the original data (see Table 4) it is necessary to determine

$$Y_{i,j} = \frac{y_{i,j} - y_{\cdot,j}}{s_j} \tag{1}$$

where $y_{i,j}$ is the response j (i.e., Cr_2O_3 grade, etc.; $j = 1, \ldots, 5$) in the treatment i ($i = 1,n$; $n = 32$), $y_{\cdot,j}$ and s_j are the related mean and standard deviation, whereas $Y_{i,j}$ is the standardized response (13). The observations $Y_{i,j}$ constitute a 32×5 matrix noted as **Y**:

$$\mathbf{Y} = \begin{vmatrix} Y_{1,1} & Y_{1,2} & Y_{1,3} & Y_{1,4} & Y_{1,5} \\ \ldots & \ldots & Y_{i,j} & \ldots & \ldots \\ Y_{32,1} & Y_{32,2} & Y_{32,3} & Y_{32,4} & Y_{32,5} \end{vmatrix} \tag{2}$$

From the matrix **Y** (32×5) it is possible to evaluate the correlation matrix **R** (5×5) as follow:

$$\mathbf{R} = \frac{1}{n} \cdot Y^{\mathrm{T}} Y \tag{3}$$

From this last matrix it is possible to evaluate eigenvalues, related eigenvectors, and scores according the standard procedure of factor analysis (11, 13, 14).

Table 6 shows the correlation matrix of the data reported in Table 4 and those obtained for the tails through consideration of the standard procedure. The eigenvalues (11) of the correlation matrix are shown in Table 7; the correspondent loadings (proportional to eigenvectors) are reported in Table 8. Figure 2 shows the projections of the scores in the two new axes Z_1 and Z_2 (13, 14). This figure, coupled with the results show in Table 7, indicate that the original 5-dimension data can be described by only one principal component. This mean that all the responses (grades components) considered in this analysis are highly correlated and therefore not independent. In particular, Table 7 shows that the major part of the total variance is explained with only one response (89.1%). This means that among the responses investigated, only one

TABLE 6
Correlation Matrix of Data in Table 4

	Cr_2O_3	Fe_2O_3	Al_2O_3	SiO_2	MgO
Cr_2O_3	1.000	0.978	0.968	−0.995	−0.653
Fe_2O_3	0.978	1.000	0.965	−0.976	−0.664
Al_2O_3	0.968	0.965	1.000	−0.969	−0.711
SiO_2	−0.995	−0.976	−0.969	1.000	0.668
MgO	−0.653	−0.664	−0.711	0.668	1.000

TABLE 7
Eigenvalues of the Correlation Matrix

Axis	Eigenvalue	Percentage of total	Cumulative percentage
1	4.453	89.1	89.1
2	0.480	9.6	98.6
3	0.036	0.7	99.3
4	0.028	0.6	99.9
5	0.004	0.1	100.0
Total	5.000	100.0	

variable is independent. From a practical point of view, we can consider the Cr_2O_3 grade as the only independent variable among all the responses considered in the experimental tests. Figure 3 was obtained by starting from the results shown in Table 4 and considering the factor analysis results. The high correlations among the grades are highlighted. For example, the presence of this correlation means that with an increase of the Cr_2O_3 composition, the SiO_2 grade decreases in a linear manner (see Fig. 3). From this observation it is possible to conclude that any combination of treatments that favors Cr_2O_3 enrichment depresses enrichment of SiO_2 in the concentrate. Analogous observations can be made for the other elements.

From the factor analysis it can be concluded that it is possible to study only the influence of the various operative factors of the MGS on the composition of only one element (for example, the Cr_2O_3 grade). This experimental evidence allows us to consider only the Cr_2O_3 composition in the concentrate for an evaluation of how the various factors influence chromite recovery in the mineral processed by MGS. Figure 3 shows the relationships among the components present in the tailing and in the concentrate fraction. A regression analysis was carried out in order to determine the correlation among all

TABLE 8
Component Loadings

	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5
Cr_2O_3	0.983	0.159	−0.058	−0.054	0.047
Fe_2O_3	0.979	0.139	−0.033	0.142	−0.005
Al_2O_3	0.984	0.063	0.166	−0.017	0.000
SiO_2	−0.985	−0.139	0.061	0.071	−0.044
MgO	−0.768	0.641	0.018	−0.000	−0.001

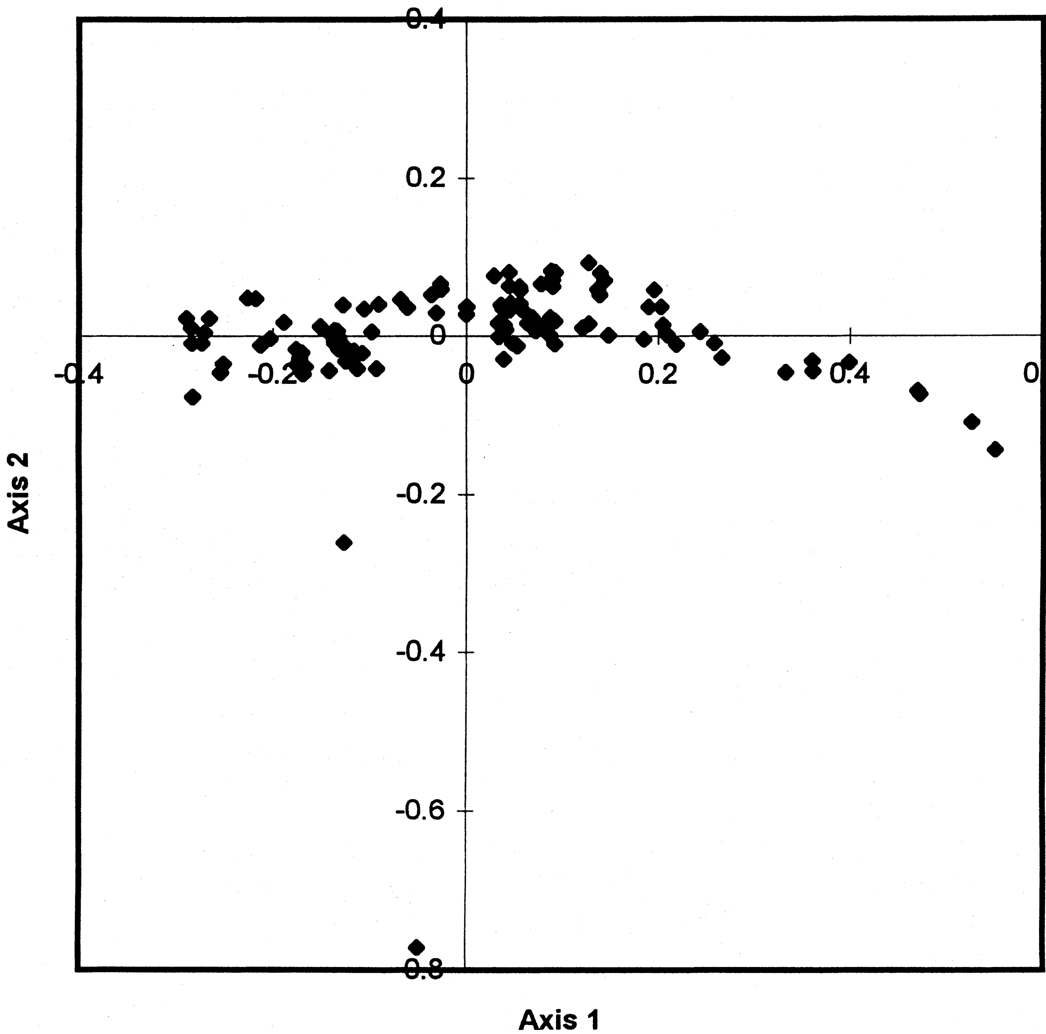

FIG. 2 Projection of the original experimental data in the new reference system: Results of factor analysis.

the grade components with respect to the Cr_2O_3 grade (Fig. 3). Similar results (elaborated by FA) were obtained by considering the recoveries (Table 5) as responses (data not shown here). In our case Cr_2O_3 recoveries were considered as the independent responses of the factorial experiment.

Based on the results obtained by factor analysis, ANOVAs were carried out only for the Cr_2O_3 grade and its recovery in the concentrate fraction. Tables 9 and 10 show the significant effects obtained from considering the Cr_2O_3 grade and recovery, respectively. From an analysis of these results it was observed that:

1. The most important effect is E (drum speed). It has a negative influence on the Cr_2O_3 grade on the concentrate fraction. The factors shake frequency,

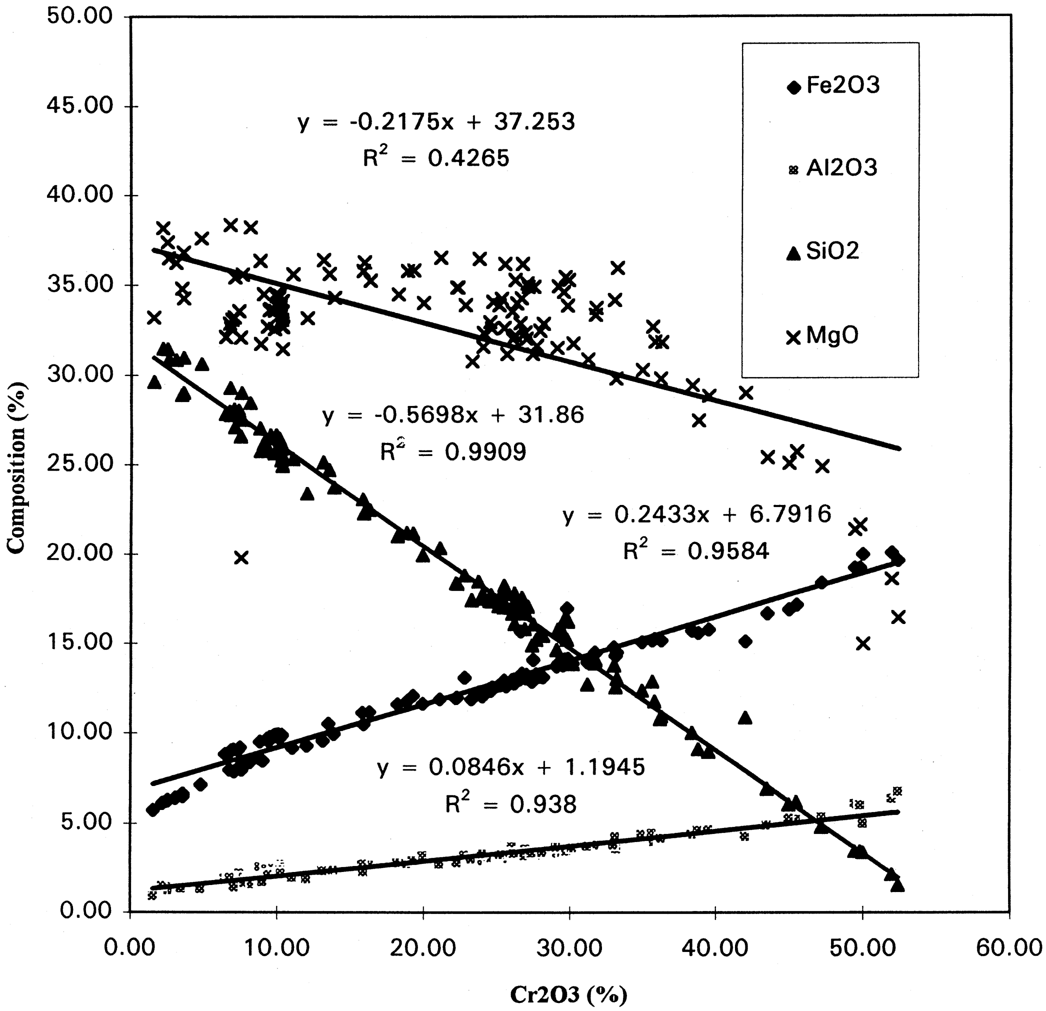

FIG. 3 Correlation among the Cr_2O_3 grades and other compounds present in the feed, concentrates, and tailings.

tilt angle and washwater flow rate (factors A, C, and D, respectively) increase the Cr_2O_3 grade in the concentrate by interacting positively (AC, BC, and BD), although factors C and D give a negative interaction ($CD < 0$). It is possible that although there is not a significant main effect of factor B (shake amplitude), this factor influences positively the Cr_2O_3 grade response with its interactions with the factors C, D, and E ($BC > 0$, $BD > 0$, and $BE > 0$). Factors A, B, C, and D interact in a different manner with respect to the drum speed (factor E): factors A and B present a positive interaction with factor E whereas factors C and D have a negative interaction with the same factor. Factor E decreases the Cr_2O_3 grade in the concentrate and presents a positive interaction with the size fraction ($EF > 0$), which

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TABLE 9
ANOVA Considering the Cr_2O_3 Grade as Response: $S^2 = 2$ with 4 *df*

Effect	%	MS	F	Significance (%)
A	1.58	20.05	10.03	97
B	1.00	8.15	4.07	89
AB	−0.87	6.13	3.06	85
C	3.95	125.09	62.54	100
AC	2.48	49.17	24.58	99
BC	2.12	35.89	17.94	99
ABC = DEF	0.66	3.49	1.74	74
D	4.31	148.30	74.15	100
AD	1.07	9.21	4.61	90
BD	1.51	18.28	9.14	96
ABD = CEF	0.97	7.48	3.74	87
CD	−2.79	62.41	31.20	99
ACD = BEF	−3.06	75.06	37.53	100
BCD = AEF	−3.36	90.42	45.20	100
EF	8.81	621.02	310.51	100
E	−18.83	2835.98	1417.99	100
AE	2.82	63.59	31.80	100
BE	2.35	44.11	22.05	99
ABE = CDF	3.66	107.05	53.53	100
CE	−2.35	44.34	22.17	99
ACE = BDF	−2.23	39.76	19.88	99
BCE = ADF	−0.45	1.61	0.80	58
DF	−1.43	16.25	8.13	95
DE	−2.82	63.42	31.71	100
ADE = BCF	−1.20	11.48	5.74	93
BDE = ACF	−1.45	16.98	8.49	96
CF	−1.24	12.33	6.17	93
CDE = ABF	−0.10	0.09	0.04	16
BF	1.38	15.25	7.62	95
AF	1.53	18.83	9.42	96
F	0.35	0.99	0.50	48

means that the negative effect of the drum speed on Cr_2O_3 grade is less negative in separation tests carried out with the largest size minerals.

2. Although the three-order interactions were initially hypothesized to be negligible, the ANOVA revealed the inconsistency of this assumption. In fact, several three-order interaction were found to be statistically significant for the responses under study. [these three-order interactions are confused with this fractional factorial design because they are a "design of resolution V" (10).] For example, the significant interaction BCD is confused with the interaction AEF because BCD and AEF are defined as *aliases*. To separate these aliases, it is necessary to perform further tests

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TABLE 10
ANOVA Considering the Cr_2O_3 Recovery as Response: $S^2 = 36$ with 4 *df*

Effect	%	MS	F	Significance (%)
A	−12.99	1350.54	37.51	100
B	−13.81	1526.72	42.41	100
AB	−4.04	130.87	3.63	87
C	−5.05	204.03	5.66	92
AC	1.30	13.61	0.37	43
BC	1.52	18.42	0.51	49
ABC = DEF	6.63	351.42	9.76	96
D	−9.28	689.28	19.15	99
AD	0.52	2.17	0.06	18
BD	−0.44	1.56	0.04	15
ABD = CEF	2.44	47.63	1.33	69
CD	3.92	122.81	3.41	86
ACD = BEF	9.09	661.52	18.38	99
BCD = AEF	10.21	833.46	23.15	99
EF	−2.18	38.16	1.06	64
E	46.78	17504.27	486.22	100
AE	8.54	584.00	16.22	98
BE	12.44	1238.92	34.41	100
ABE = CDF	3.00	72.06	2.00	77
CE	7.20	414.99	11.52	97
ACE = BDF	1.79	25.79	0.71	55
BCE = ADF	−0.36	1.08	0.03	13
DF	−2.24	40.33	1.12	65
DE	3.87	119.78	3.33	86
ADE = BCF	−2.69	58.03	1.61	73
BDE = ACF	−3.56	101.49	2.82	83
CF	−5.76	265.11	7.36	95
CDE = ABF	−5.09	207.30	5.76	93
BF	−10.82	937.40	26.03	99
AF	−8.32	554.02	15.39	98
F	46.65	17412.55	483.68	100

utilizing the projection property of the fractional factorial experiments (10). Because of the aim of this preliminary work, we preferred to separate these aliases by physical considerations by considering the significant main and two-order interaction effects previously reported (10). For example, because interaction BCD is confused with AEF, we are unable to determine what kind of interaction could be significant. In this case it is very probable that interaction AEF is a significant interaction because the ANOVA has shown a large effect by the interaction EF (10). The presence of interaction AEF (−3.4%) means that the positive effect of interaction EF (+8.8%) decreases in tests in which the highest level of factor A (shake frequency) is considered. By using similar considerations, we tried to discriminate the other aliases of the three-order interactions. In this manner we obtained:

$$ACD = BEF \quad \rightarrow \quad BEF \text{ as significant}$$

$$ACE = BDF \quad \rightarrow \quad BDF \text{ as significant}$$

$$ABE = CDF \quad \rightarrow \quad CDF \text{ as significant}$$

3. In general, factors that improve the Cr_2O_3 grade decrease its recovery and vice versa (7, 12). For example, the drum speed (E) decreasing the Cr_2O_3 grade into the concentrate but increases its recovery in the concentrate. In the same manner, the main effect of the shake frequency, shake amplitude, and washwater flow rate factors (A, B and D, respectively) decrease Cr_2O_3 recovery. At the same time, there are positive interactions among factors A, B, and C with factor E. In other words, there is a minor negative effect on Cr_2O_3 recovery by A, B, and C when the drum speed is set at its highest level. The mineral size factor (F) has a positive effect on Cr_2O_3 recovery. It is possible that this is because the best Cr_2O_3 grade occurs in the largest size fraction of the mineral. However, this result indicates that enrichment of chromite in the concentrate is favored for the largest mineral size. Factors A and C present a negative interaction with F, indicating that the negative effects of factors A and C are more negative for chromite recovery in tests carried out with the largest size fractions.
4. For Cr_2O_3 recoveries in the concentrate, some three-order interactions were found to be significant. Using the same considerations reported above, we discriminated the aliases of the three-order interactions as follow:

$$ACD = BEF \quad \rightarrow \quad BEF \text{ as significant}$$

$$BCD = AEF \quad \rightarrow \quad AEF \text{ as significant}$$

For both ANOVA (Cr_2O_3 grade and recovery in the concentrate), the presence of these three-order interactions indicates a more complicated and intrinsic relationship among the process parameters of MGS. At the moment, no physical conclusions can be reached from an analysis of these interactions.

In conclusion, the empirical models (10) obtained by the fractional factorial experiments and by ANOVA can be used to estimate MGS performance in the investigated range of experimental conditions. These models need further improvement due to the presence of three high-order interactions. Therefore, the more probable significant three-order interactions were selected by performing the assumption reported above. If there is confusion [for example, between interactions BCD and AEF (see Table 9)], the three-order interaction in which there are a major number of significant two-order interactions with the same factors present has been considered significant.

This degree of approximation was considered acceptable at this stage of the work. Further work is in progress to describe the Cr_2O_3 grade and recovery using the surface response methodology (8).

It is possible to use the empirical model related to the factorial experiment as a first approximation in order to calculate the Cr_2O_3 grade and recovery in the concentrate and to perform an analysis of the residues to check the assumptions on the experimental error distribution of the factorial designs (10). The parameters of these empirical models are the significant effects reported in Tables 9 and 10. The independent variables (i.e., drum speed, tilt angle, etc.) are transformed into coded forms (8, 10). For example, the levels of the variable X_5 (E = drum speed) in coded form can be written as

$$X_5 = \frac{\text{rpm} - 195}{35} \quad (4)$$

The statistical models for the Cr_2O_3 grade and recovery, respectively, are reported (see Tables 9 and 10 for the effect values) (10) as

$$Y_1\ (\%) = 33.8 + (1.58X_1 + 3.95X_3 + \cdots + 8.81X_5X_6)/2 \quad (5)$$

$$Y_2\ (\%) = 51.9 + (-12.99X_1 - 13.81X_2 + \cdots + 8.81X_5X_6)/2 \quad (6)$$

where Y_1 is the Cr_2O_3 grade in the concentrate (%); Y_2 is the Cr_2O_3 recovery in the concentrate (%); X_1, X_2, X_3, X_4, X_5, and X_6 are the values of the factors that can be set in the MGS, (Factors A, B, C, D, E, and F, respectively) expressed in coded form (−1; +1).

In Figs. 4 and 5, the Cr_2O_3 (grade and recovery, respectively) experimental vs calculated values of these empirical models are reported. The results of this analysis show an acceptable fitting of the statistical model of the factorial experiment. Moreover, an analysis of the residues (not shown here) showed that the normal distribution of the experimental error was an acceptable assumption (8, 10).

Considering the results reported above, it is possible to establish the following procedure to estimate the performance of a MGS:

1. By using factorial models containing all the significant effects of the MGS process factors, the Cr_2O_3 grade and its recovery can be estimated by setting the process parameters.
2. It is possible to evaluate the Cr_2O_3 grade and recovery in both the concentrate and tail by using a material balance.
3. By using the empirical correlation shown from factor analysis (Fig. 3), it is possible to determine the component concentration in each fraction and complete the material balance;

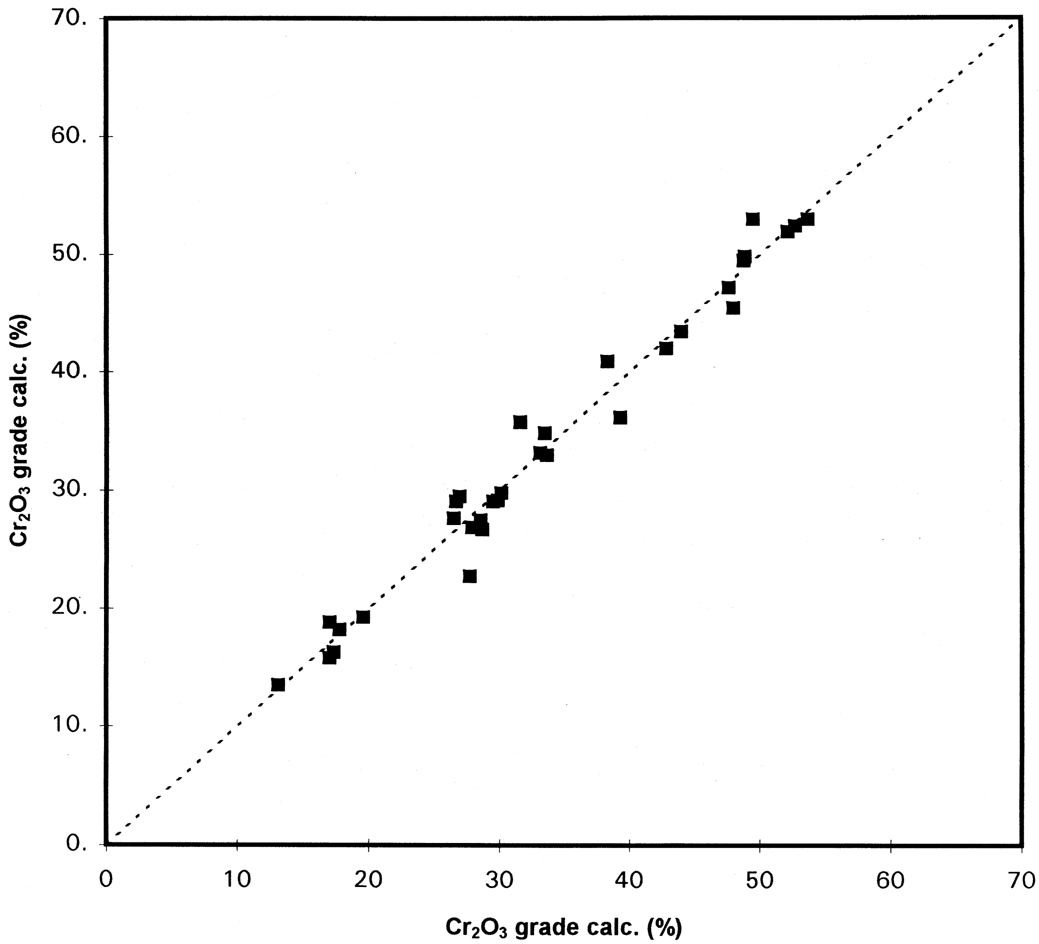

FIG. 4 Cr_2O_3 grade in the concentrate fraction calculated by Eq. (5) vs experimental.

A limitation of these models and procedures is due to their empirical nature. They will not be applicable in any situation other than one strictly comparable with the experimental condition used in the present work. In other words, scale-up problems could be a very important feature to be considered in further studies for this apparatus.

Further Considerations

The separation or the enrichment factor of chromite fines may be evaluated by the Cr_2O_3/SiO_2 ratio. From factor analysis and from the analysis of Fig. 3 it is possible to observe that by improving the Cr_2O_3 grade in the concentrate through manipulating the machine's parameters, a decrease of SiO_2 grade can be obtained in this fraction. Figure 6 reveals this kind of relationship. The Cr_2O_3/SiO_2 ratio of the two fraction sizes utilized in the experimental tests were 0.39 and 1.49, respectively, for the smallest and largest mineral sizes.

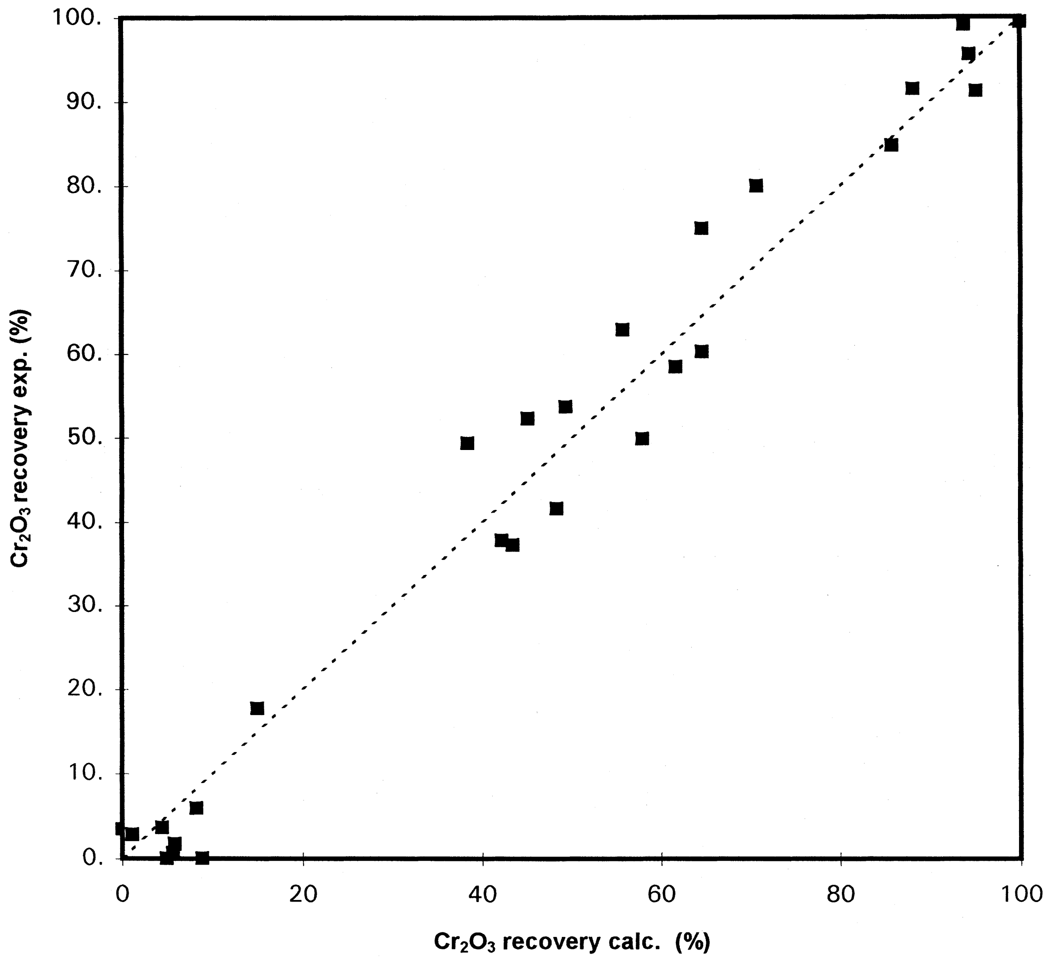

FIG. 5 Cr_2O_3 recovery in the concentrate fraction calculated by Eq. (6) vs experimental.

Considering that Fig. 6 reports all the data obtained for the two mineral fractions the tails and the concentrates it is then possible to observe that for a given Cr_2O_3/SiO_2 ratio and Cr_2O_3 grade the separation process splits this value into two different values of the Cr_2O_3/SiO_2 ratio and then of the Cr_2O_3 grade for the sterile and concentrate fractions. This diagram shows the limits for each fraction in terms of selectivity of the separation that can be achieved under the investigated experimental conditions.

Another aspect that must be considered is the relationship between Cr_2O_3 recovery and the recoveries of the other component (i.e., SiO_2, Fe_2O_3, etc.). As an example, Fig. 7 reports the relation between Cr_2O_3 recovery and SiO_2 and Fe_2O_3 recovery. In this case it is possible to observe that the best experimental condition for the separation of Cr_2O_3 and SiO_2 is obtained Cr_2O_3 recovery when ranges from 60 to 80%. In this condition a rather large distance exists between the SiO_2 recovery curve and the Fig. 7 diagonal. An increase

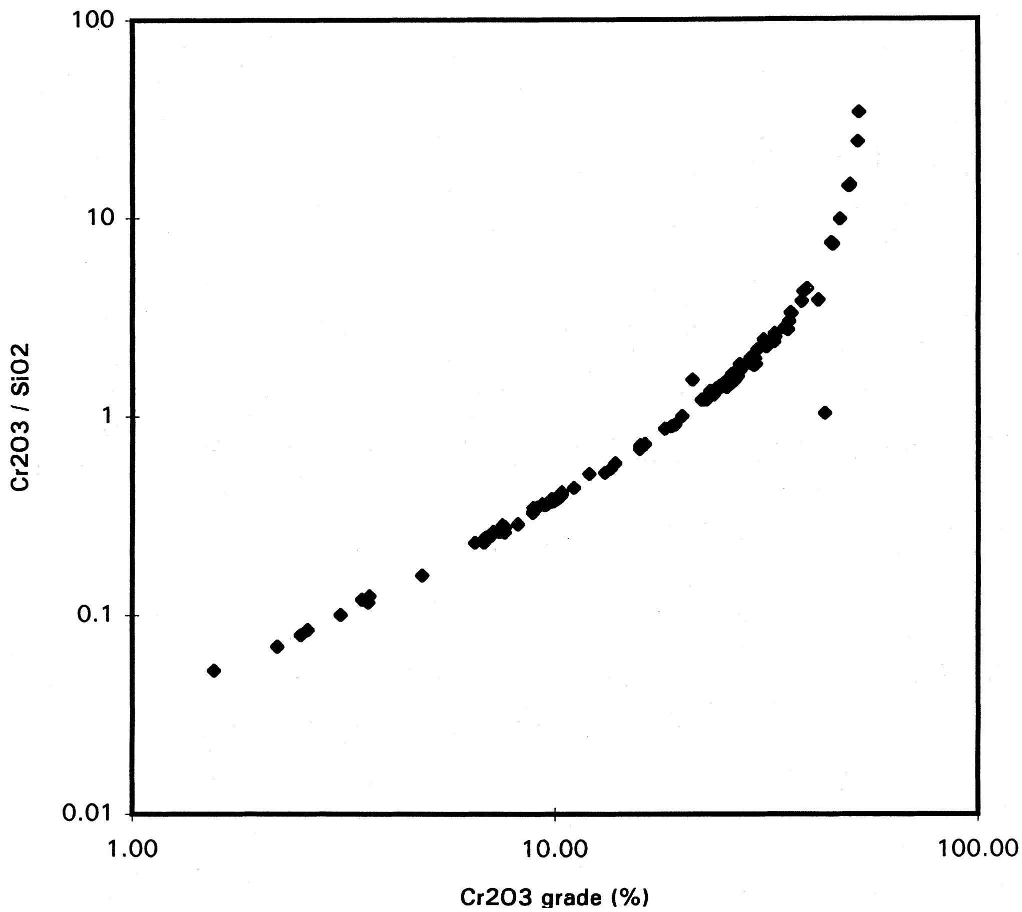

FIG. 6 Relationship between the Cr_2O_3 grade in the feed, concentrates, and tailings and the Cr_2O_3/SiO_2 ratio (w/w).

in Cr_2O_3 recovery to more than 95% produces a rapid decrease in selectivity. The minor difference observed between Cr_2O_3 and Fe_2O_3 recoveries is linked to mineralogical analysis of the mineral because they are both present in the olivine (7).

Figure 8 shows the relation between Cr_2O_3 recovery and its grade in the concentrate for different size mineral fractions. The empirical equations shown in Fig. 8 were used in the experimental data fitting. In this case it is possible to highlight the behavior of Cr_2O_3 for the two sizes of mineral fractions. The upper curve is for the behavior of the largest grains whereas the lower curve is for the smallest grains. Analysis of these results shows.

- It is possible to reach large Cr_2O_3 grades but with low recoveries in both cases.
- As reported in the literature, enrichment of fine fractions is more difficult. In this case the maximum Cr_2O_3 recovery in the concentrate fraction is about

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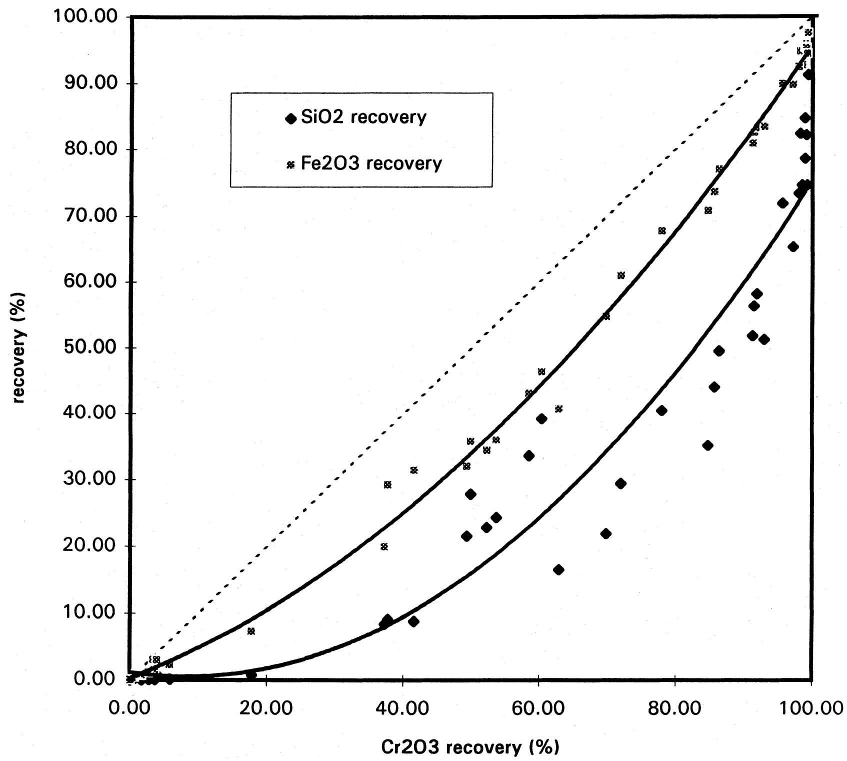

FIG. 7 Dependence of SiO_2 and Fe_2O_3 recovery on Cr_2O_3 recovery.

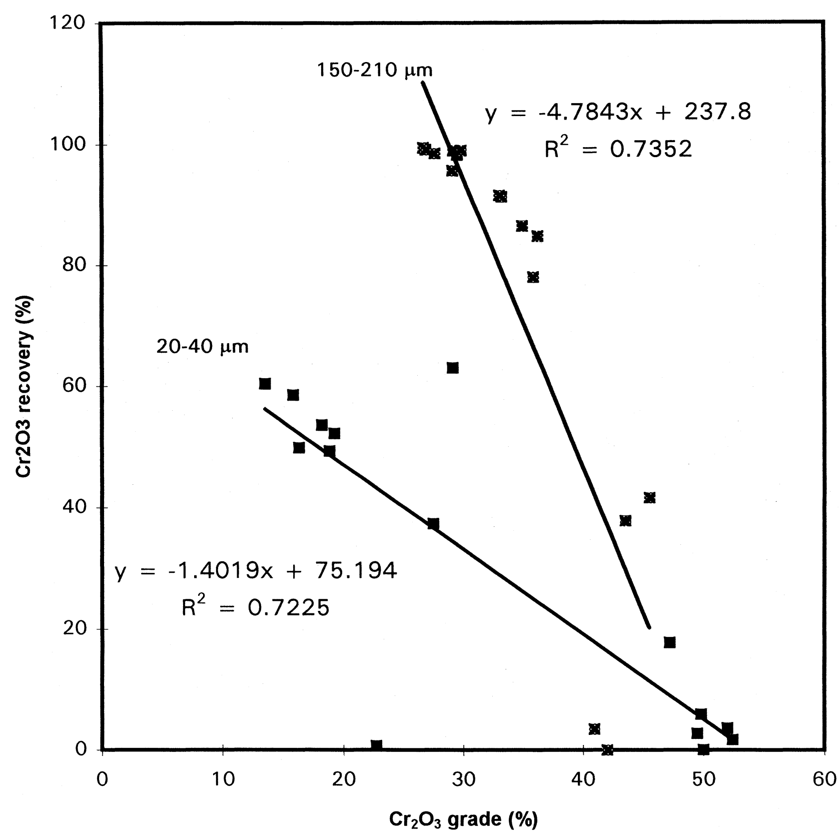

FIG. 8 Cr_2O_3 recovery vs Cr_2O_3 grade for different size fractions of the ore.

60%. This difficulty is also indicated by the slopes of the curves. A variation in Cr_2O_3 grade with fine fractions produces a smaller variation in Cr_2O_3 recovery based on experimental tests carried out with the larger size fractions.

Analysis of these results shows that it is possible to establish a compromise between the Cr_2O_3 grade and its recovery by, for example, an economic analysis.

CONCLUSIONS

A fractional factorial experiment was performed in order to establish the effects of some processing factors on the Cr_2O_3 grade and recovery into the concentrate obtained by MGS fed with chromite fines. The study showed how the factors that improve the Cr_2O_3 grade also decrease its recovery in the concentrate. The high-order interactions found by ANOVA indicate that any optimization study must consider all the operative variables at the same time. Factor analysis permitted us to find a strong correlation among the component grades present within the concentrate and sterile fractions. This fact permitted us to consider only the ANOVA utilizing Cr_2O_3 grade as a determinant of the process because of its correlation with the other component compositions. The results in this paper reported on a methodological approach to a study of the performance of a multigravity separator for the concentration of fine ores and, in general, for multifactor experiments with multiresponses.

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